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Isogeometric analysis and form finding for thin elastic shells

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Abstract

Recent developments within the design of shells have seen an increased interest in utilizing active bending as form giving procedure [1]. This enables complex structures to be built from simple off-the-shelf materials. However, forming bending-active structures is highly dependent on the material properties, which makes the design process reliant on either physical testing or digital simulations. An associated problem with the simulation of this behavior is the lack of integration between modeling and analysis in conventional simulation techniques, a crucial concern since the final design is always an equilibrium shape with requirements on both structural and spatial integrity. IsoGeometric analysis (IGA) is a method that aims to bridge precisely that gap between analysis and design, making it a suitable method for bending active structural design. This thesis explores an approach to the modeling and digital design of actively bent shells using the implementation of nonlinear IGA. Further on, two different ways of controlling the geometry, either by tracing the process *forwards* or *backwards* during the construction procedure are proposed. Tracing the process forwards returns an implicitly controlled shell geometry through stepwise displacement of the boundaries of a flat sheet. However, as a design approach, one is often interested in explicitly controlling the final geometry by a *backwards* tracing. This allows the designer to start from a desired outcome and instead tailor the material to approximate this desired form. The procedure is tested in a case study where a combination of both *forwards* and *backwards* tracing is included. Both processes apply the Kirchhoff-Love shell theory [6] and uses the total Lagrangian formulation for the nonlinear computations.

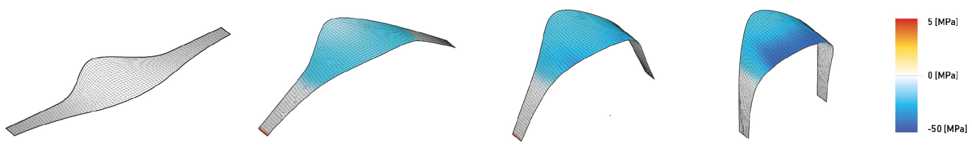

Figure 1: Process of forming a flat sheet (l) into a predefined form (r) showing stress in the bottom fibre.

Keywords: Isogeometric analysis, form finding, active bending, shell design

1 Introduction

The initiative behind this paper lies in an interest to continue exploring the integration of structural design and architectural geometry, and furthermore the application of elastic bending deformation as a form giving procedure. Isogeometric analysis (IGA) is the main technique applied including a non-linear Total Lagrangian (TL) -formulation. The main body of the work can be found in this master thesis project[4]. The first part of the paper gives a short introduction to IGA and the concept of active bending including the specific numerical methods applied. The second part describes the implementation of two different techniques for how to integrate IGA based active bending in a design context to assist in the search for efficient shell structures. The modes of controlling the outcome of nonlinear elastically deformed shell geometries are illustrated by a test case. Finally a discussion on the potential of these methods, how they can be combined, what limitations have been identified and a recommendation of further work is presented.

1.1 Isogeometric Analysis

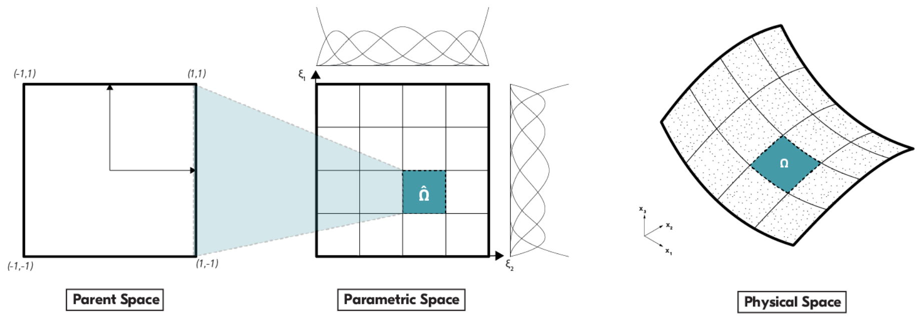

Figure 2: Illustration of the three spaces for a 2D surface that are essential to the IGA configuration.

In Isogeometric Analysis the shape function that describe the distribution of the unknown field variable for the analysis is chosen as the same shape functions that determine the geometry of the object. The most common implementation takes the shape functions of NURBS as point of departure. Isogeometric Analysis was introduced by Huges et. al in 2004 and has since gained a lot of attention [11]. Some implementations into the field of structural design and architecture can be found in [10], [6], [1]. The basic equation behind the shape functions used for NURBS modelling reads,

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \tag{1}$$

and is solved using a recursive algorithm, such as Cox-De Boor [8]. For the application of IGA on 2D-surfaces these shape functions are applied in the two parameter direction, each associated with a control point, with the description for the surface S being obtained by a multiplication of the two such that,

$$S(\xi_1, \xi_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi_1) M_{j,q}(\xi_2) P_{i,j} \tag{2}$$

where n, m are the number of control points in direction 1 and 2, N, M are the shape functions in the two directions and $P_{i,j}$ is the bidirectional control point net. Figure 2 shows the 3 spaces used for the Isogeometric formulation. Compared to the standard FE-formulations the main difference with IGA is in the inter-continuity of elements in the parametric space as the shape functions are defined on clusters of elements, referred to as patches. This paper will however only cover the application of single patches.

1.2 Active Bending

The term *Bending active* was coined by Julian Lienhardt [7] and refers to free form structures which get their shape from the deformation of initially planar elements, as shown in Figure 3. It is an attractive approach because of its possibilities to create complex shapes from relatively simple and cheap materials. Similar principles have been used in the design of the Mannheim Multihalle gridshell, the Japaneese Pavillion and the Downland gridshell, where the final shape is achieved by pushing the boundaries together, starting from a flat initial position [9]. However, these examples are multilayered structures where the joints are lose during erection and tightened when the final position is reached allowing for the layers to shear relative to each other as the curvature is increased. The bending active principle can be applied in component level, solar shading blindes etc. [5], [7], [3] and for full scale building such as the examples mentioned.

The construction procedure makes bending active structures difficult to design. The final geometry is a result of large deformations, tying the form, material and construction closely together. The final shape also need to be found as an equilibrium shape, based on the initially planar member. The topic has been subject to much research recently [3] including two workshops at the Advances in Architectural Geometry conference, AAG 2018.

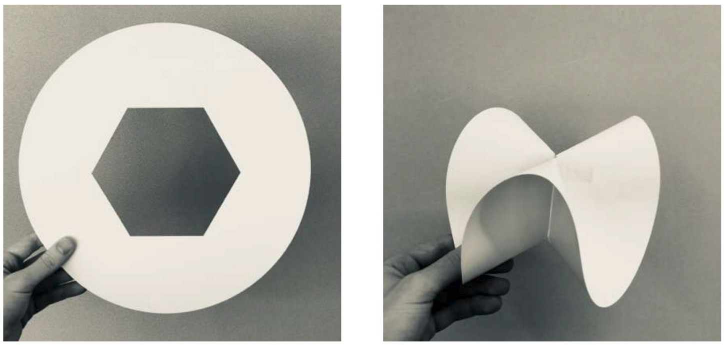

Figure 3: Example of deforming a simple flat cut out into a complex 3-dimensional shape

2 Numerical strategy

Given the large displacements inherent to the design process of bending active structures, a geometrically nonlinear formulation is needed, and since the ambition is to model behaviour of thin shells, Kirchhoff-love theory is regarded a suitable element formulation. The material is assumed to behave elastically, the strains are assumed being small. The total Lagrange formulation, with the second Piola Kirchoff stress tensor and associated Green-Lagrange strain tensor, is used to reference the geometry at time t back to it's initial state at t_0. (As opposed to the updated Lagrangian process which relates the strains back to the previous step) [2]. At any time t, the deformation is traced back to an initial time 0, cf. Figure 4. The state at time ***0*** is

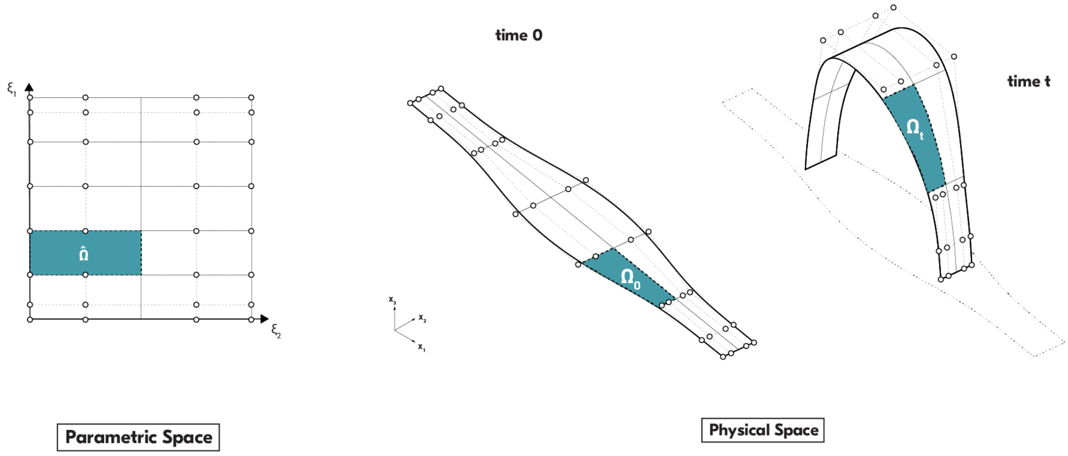

Figure 4: States of Nonlinear analysis

referred to as the reference configuration and is assumed to be represented by the geometry in its undeformed state. The state at time $\boldsymbol{t}$, denotes an arbitrary known step, from which the next and unknown step $(\boldsymbol{t}+\Delta\boldsymbol{t})$ can be computed using the displacement controlled Newton-Rapson (NR) method. For further details the reader is referred to [2] and [4].

Using the total Lagrangian formulation allows for the strains and subsequent internal forces to be computed on any geometry at time t, independent of history. The analysis can therefore be started from any position at any time $\boldsymbol{t}$, given the existence of a undeformed reference geometry.

3 Proposed Design Method

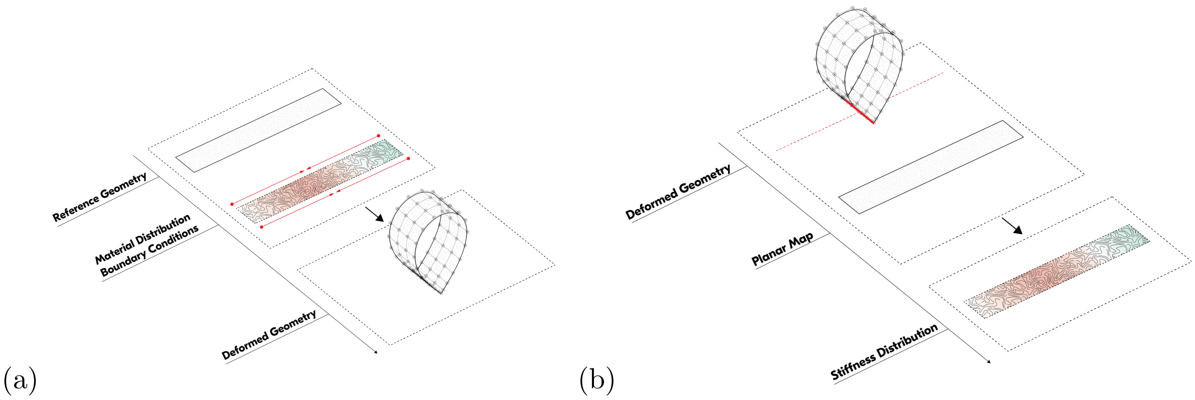

Figure 5: Diagrams showing input/output for (a) *forwards* vs. (b) *backwards* tracing.

To use the nonlinear formulation as a basis for the design method, one needs to manipulate between the undeformed and deformed geometry. The deformed geometry represents the final design, however, whether this is explictly controlled or an implicit result of the process lies in how the process is traced. If the process is traced in a timewise *forwards* manner the deformed geometry will follow from the reference geometry and the material. Numerically the process can, however, also be traced backwards, instead starting from the desired outcome as an input which enables an explicit control of the outcome. The control of the relationship between deformed and undeformed will lie in the application of a custom thickness distribution for the flat sheet, defined in the parametric space of the formulation, described as an independent NURBS parameterisation.

Forwards Tracing
In the *forwards* tracing the process, the reference geometry (time ***0***) and analysis setup (material map and boundary conditions) serve as the starting point, which is illustrated in Figure 5(a). By tracing the progression in a time-wise forwards process, the form finding will follow the same steps as a potential construction procedure, where an undeformed geometry is deformed into a new configuration. This implicitly controlled form will therefore have a clear physical interpretation. This process follows that of a conventional displacement controlled, nonlinear solution procedure and is shown in Figure 6

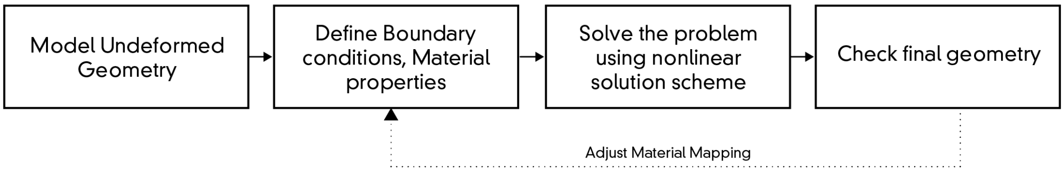

Figure 6: Flowchart for tracing the process *forwards*

Backwards Tracing
Tracing the geometry *forwards* gives an insight to the physical process of construction, but as a design exercise, however, one might want to steer the process towards a desired outcome. Thus, instead of tracing the process *forwards*, one can instead use a modeled reference form at a time ***t*** and trace the process *backwards*, by putting the desired geometry in equilibrium using the stiffness. By starting from time t, the designer gains a level of explicit control of the final geometry, instead using the material distribution as the unknown. The process can be broken down in the steps illustrated in in Figure 7:

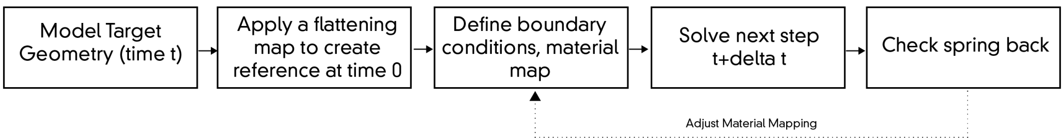

Figure 7: Flowchart for tracing the process *backwards*

3.1 Case Study (Example)

A practical modeling exercise using the aforementioned processes is illustrated in the following case study. It will recreate a modeled design geometry as a physically informed object, thus, embedding stresses and material as part of the design. The process will start from a target form and trace that backwards into a material map, which will further be used to simulate the construction and check correspondance with the desired form. A reference geometry is designed using NURBS, which can be seen in Figure 8.

The process is structured as follows:

1. Model target geometry.
2. Apply flattening map to create a reference configuration.
3. Create a material parameterisation, (in this case NURBS parameterisation).
4. Solve from step t to $t + \Delta t$.
5. minimize "spring back" through adjusting material mapping (in this case done using genetic algorithms).
6. Use the adjusted material map and flat geometry to find an approximation of target geometry.

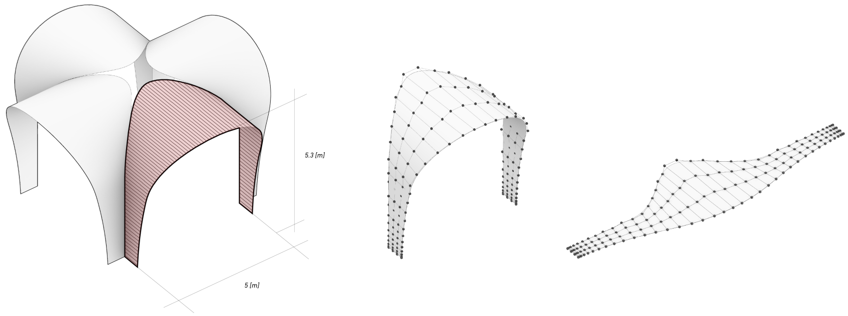

Figure 8: Modeled Design Geometry for the case application (left), Segment (middle), Flattening map (right)

Backwards Step

Going backwards, the process starts with the unrolling of the target geometry into a flat representation. From that, a material distribution is used to minimize the spring back, thus, putting the design geometry in equilibrium. The shell sheet material is parameterized using 2nd degree NURBS polynomials with a domain of thickness 2cm - 14cm, and the minimization is carried out by adjusting the control points of the thickness map using a genetic algorithm. The final material map can be seen in Figure 10. A negotiation is present here between the complexity of the material parameterisation and the final deviation.

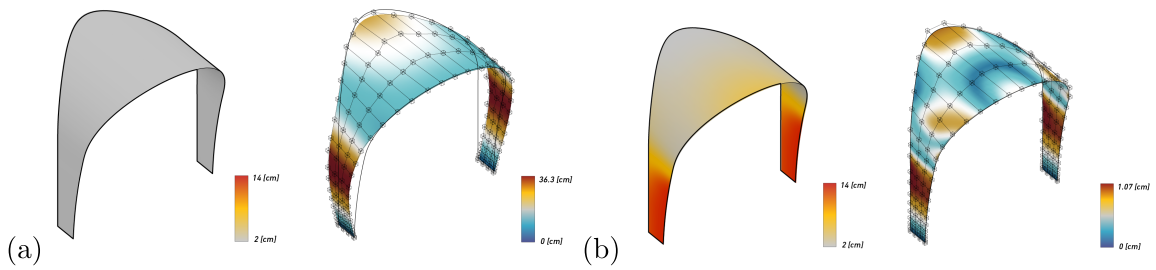

Figure 9: a)Constant thickness distribution (left) Spring back (right) b) Adjusted thickness distribution (left) Spring back (right)

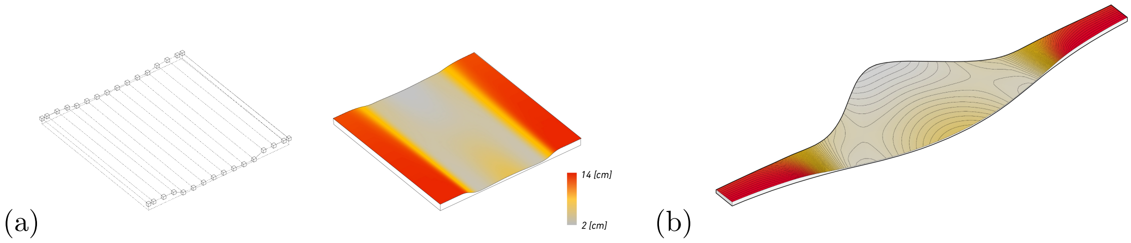

Figure 10: a) Thickness Map b) Distribution on Reference Geometry

Forwards Step

With the acquired material map, the simulation can now be performed in a forwards fashion, which should simulate the construction process and give an indication on how close to the intended geometry the design will come. Further this checks that the desired equilibrium is the one, reached, in the case of multiple possible equilibrium states. The form finding is accomplished by displacing the both ends and pushing them together, without any additional constraints. Some intermediate steps can be seen in Figure 11 a) with the final form of a segment illustrated in Figure 11 b). The final design and stresses are further shown in Figure 12 .

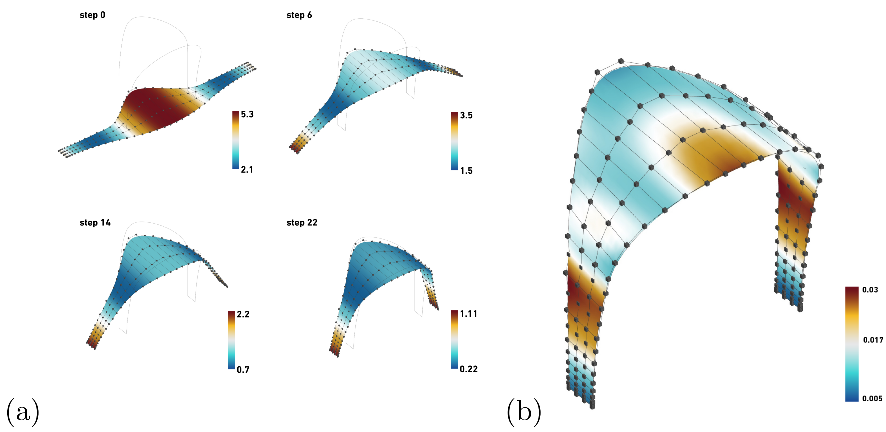

Figure 11: Form finding process / final design. Colors show deviation from target geometry in meters.

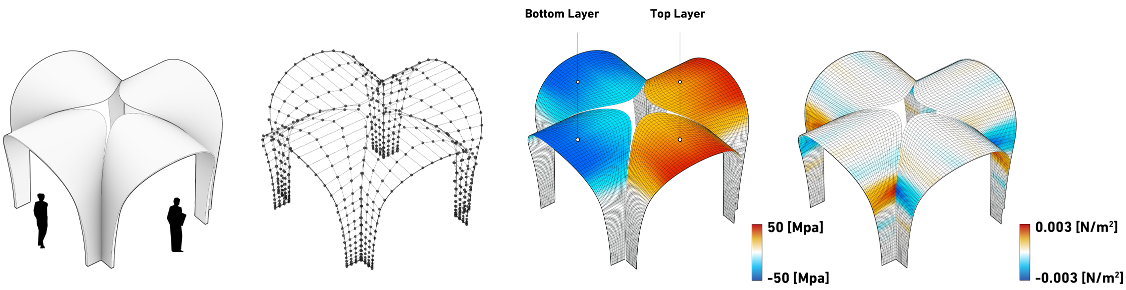

Figure 12: (from left to right) Final form, Final control polygon, Normal stress, Torsion moment

4 Conclusions and Further Work

In contrast to what is mainly associated with the term *form finding*, which is the generation of a form w.r.t a specific load case (e.g gravity) the process presented here instead targets the "load case" of the construction process, and how actively bent geometries can be controlled through the relations between *undeformed geometry*, *boundary conditions and material* and the *final design* as a mix of explicit geometric steering of the nonlinear process. The application of isogeometric analysis opens for new ways of treating the integration between digital modeling and analysis and the paper has shown one application of the integration of physical behaviour and material properties in a modeling context, but there's more potential. The process targets segmented shells and is therefore limited to the use of single patch analysis and further work is needed for multipatch models. As the material map is simply computed using genetic algorithms, the process of acquiring a suitable material distribution for a specific outcome could also be subject to further work.

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